

Proportions and Scale

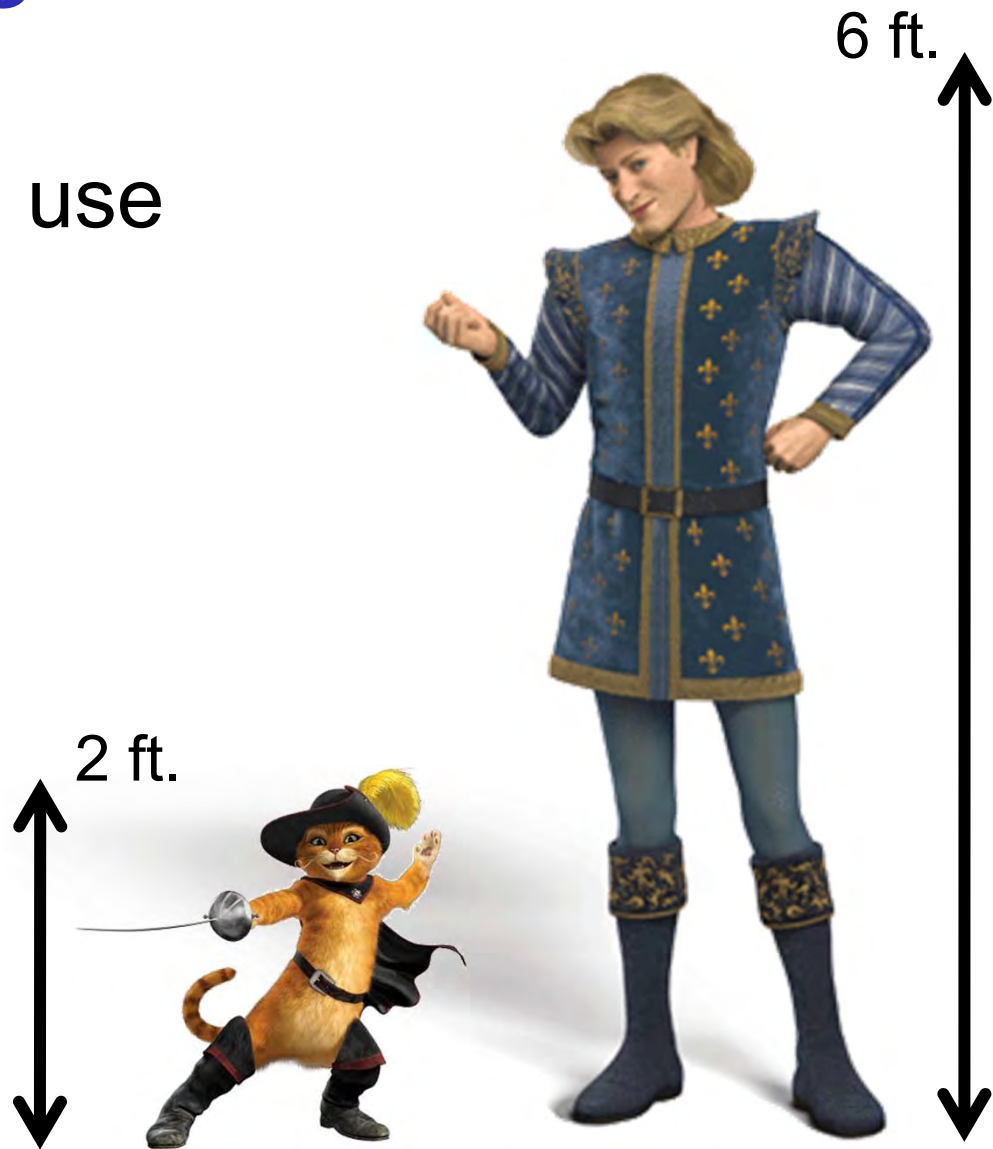


National Science Foundation
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Defining Size

For simplicity, we'll use length (or height) to define size.

For example,
Charming is 3x
the size of Puss.



Size, Area, and Volume

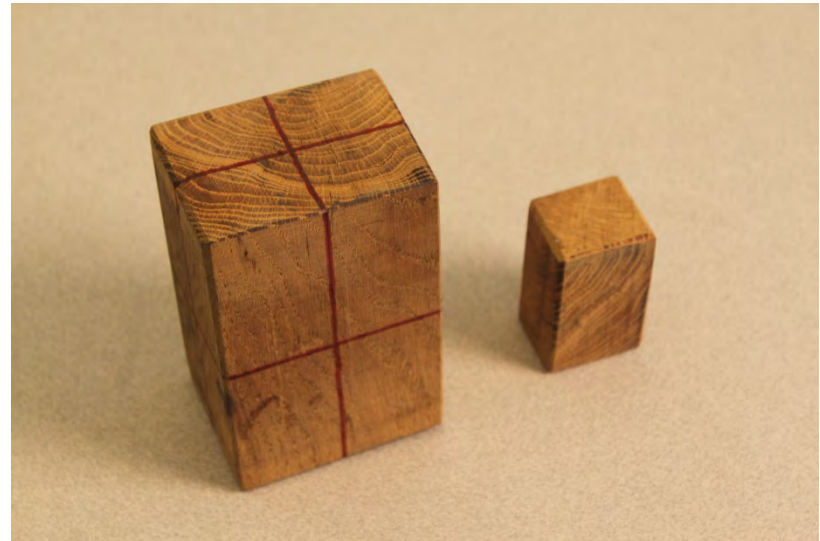
Area goes as:

$$(\text{Size}) \times (\text{Size}) = (\text{Size})^2$$

Volume goes as:

$$(\text{Size}) \times (\text{Size}) \times (\text{Size}) = (\text{Size})^3$$

Big cube is 2x the size
so it has 4x the area
and 8x the volume.



Size, Area, and Volume

Proportions of area and volume apply to any shape.

For example, to make this flower pot 3x bigger takes 9x more paint and 27x more clay.

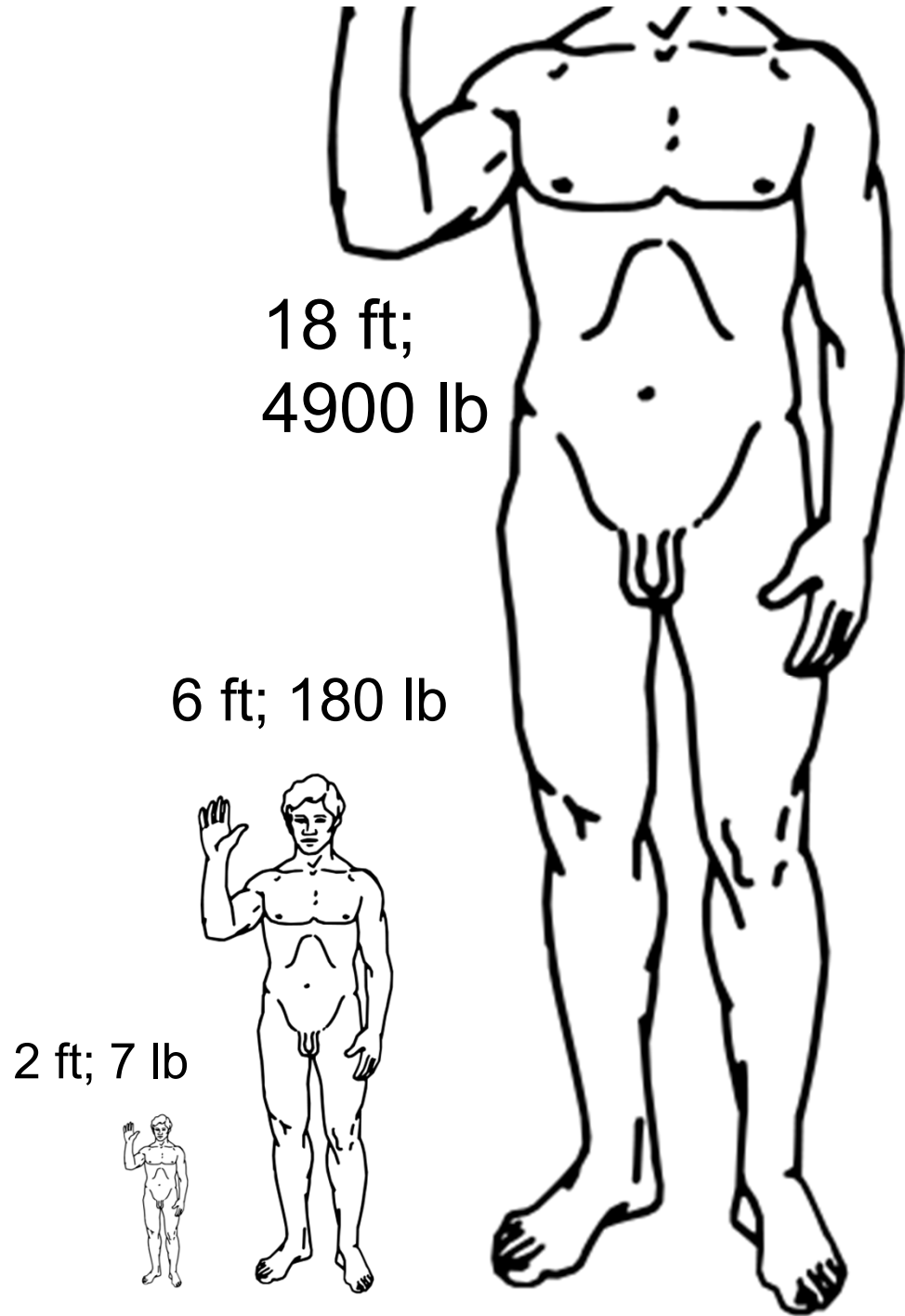


Size	Area	Volume
x $\frac{1}{4}$	x $\frac{1}{16}$	x $\frac{1}{64}$
x $\frac{1}{3}$	x $\frac{1}{9}$	x $\frac{1}{27}$
x $\frac{1}{2}$	x $\frac{1}{4}$	x $\frac{1}{8}$
x 2	x 4	x 8
x 3	x 9	x 27
x 4	x 16	x 64
x 5	x 25	x 125

Body Weight

Body weight is proportional to volume.

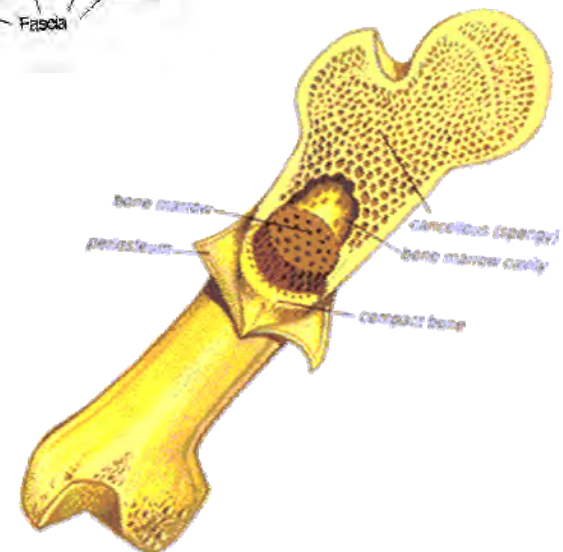
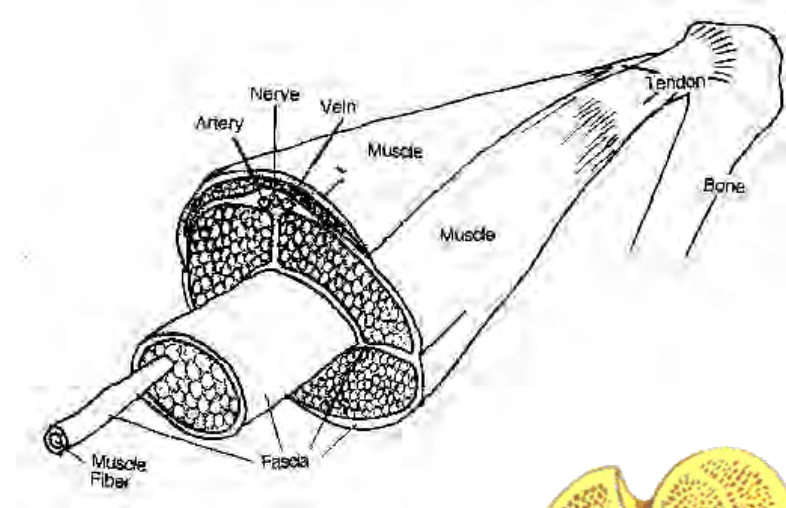
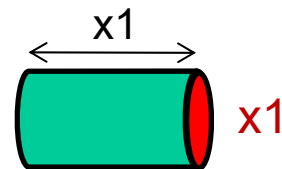
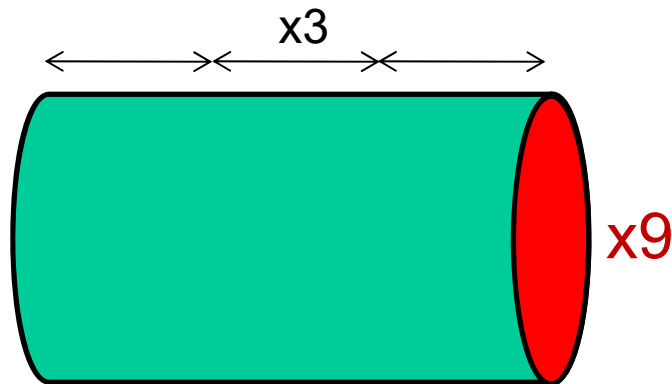
When size x3,
body weight x27



Muscle and Bone Strength

Muscle and bone strength are proportional to area.

When size x3,
strength x9



Weightlifting

Weightlifting (relative to body weight) is a physical cue for size.

Muscle force scales as area while body weight scales as volume.

Lifts 90 lb;
50% of
body weight

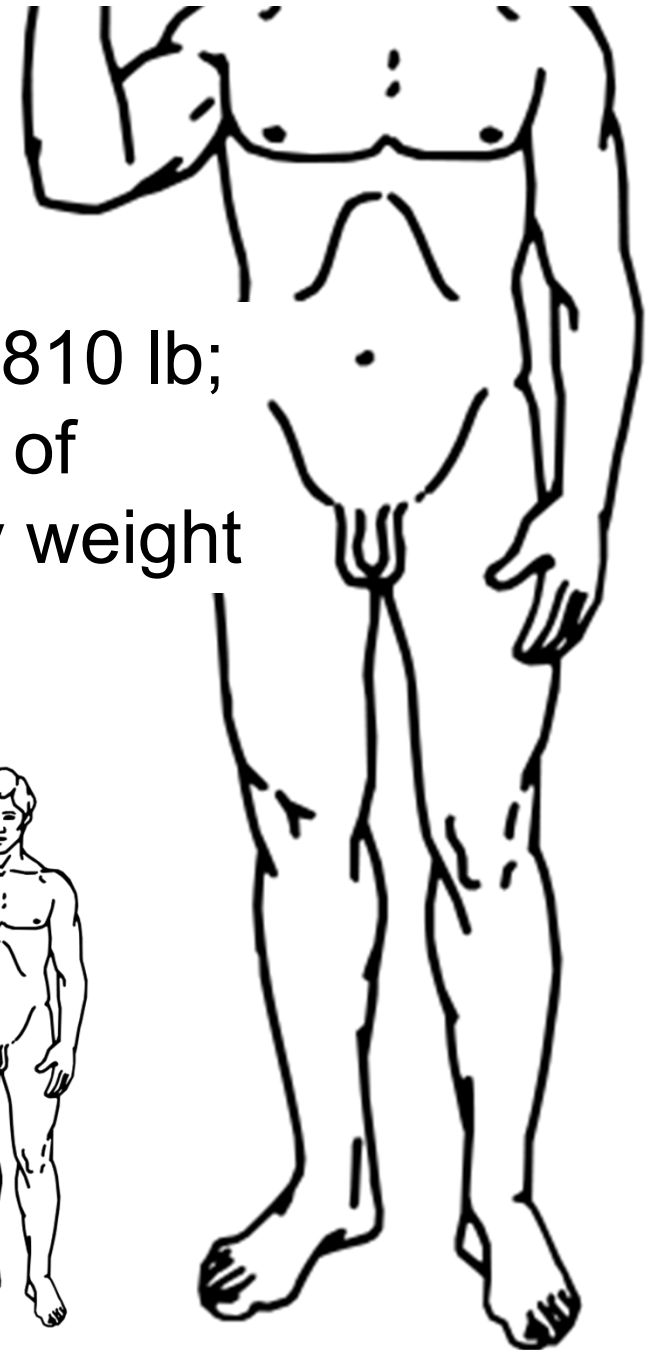
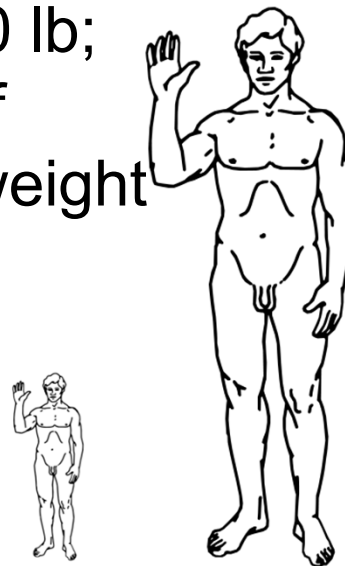
Lifts 10 lb;
150% of
body weight

Lifts 810 lb;
17% of
body weight

2 ft, 7 lb

6 ft, 180 lb

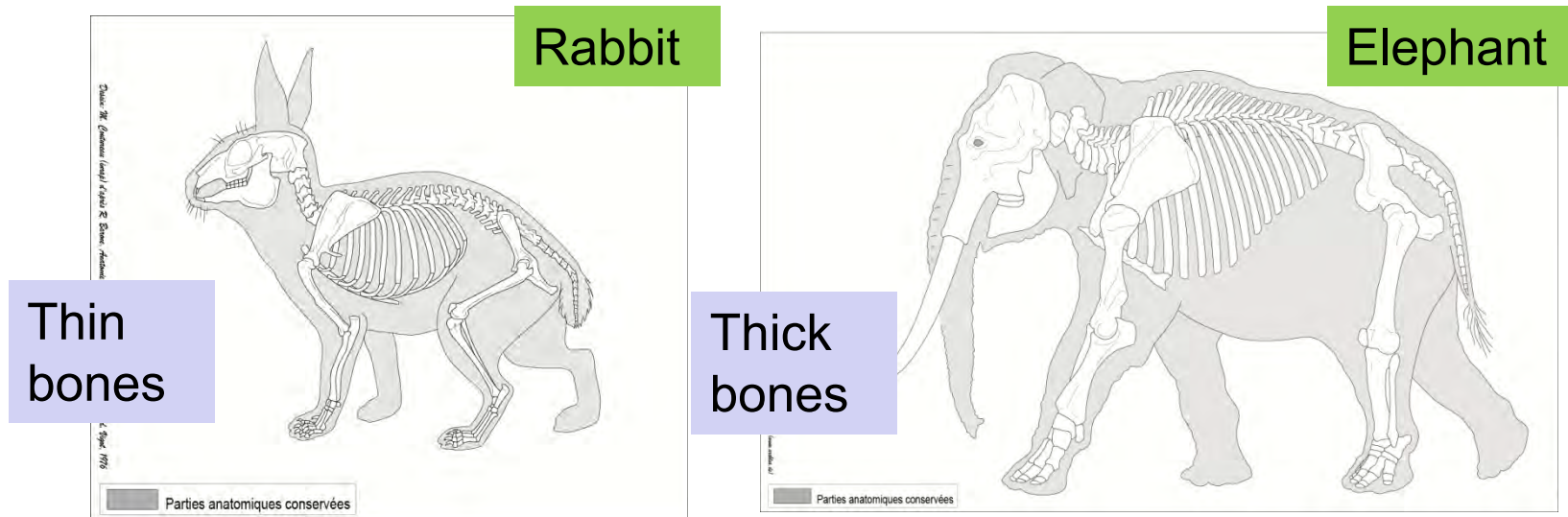
18 ft, 4900 lb



Skeletal Fraction

Skeletal fraction is a physical cue for size.

Bone strength scales as area while body weight scales as volume.



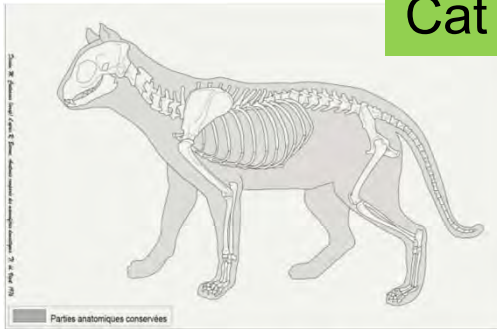
Skeleton is 9% of total weight for rabbit and 27% for elephant (their size ratio is 15-to-1)

Size and Posture

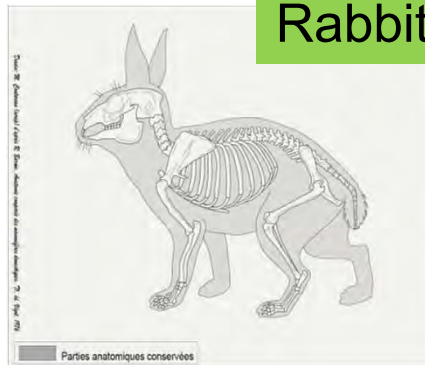
Posture, measured as mechanical advantage of limbs, scales as size

Posture is a physical cue for size.

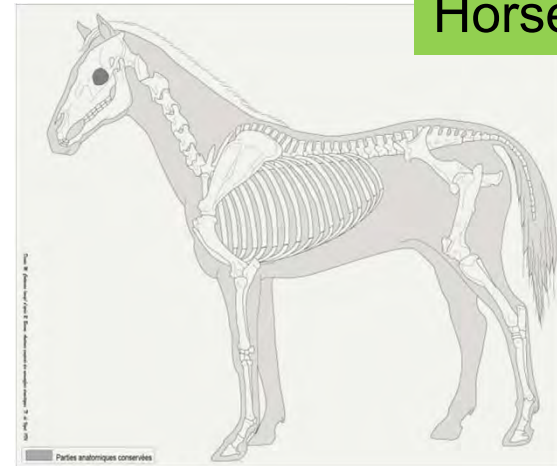
Cat



Rabbit

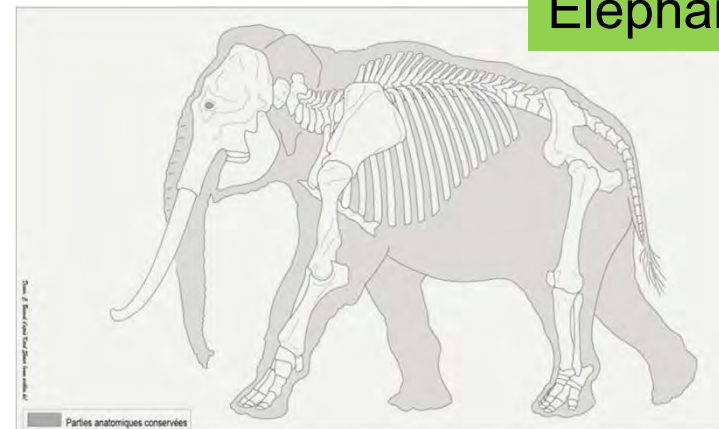


Horse



Large animals need to stand straight so as to minimize strain on bones due to weight.

Elephant



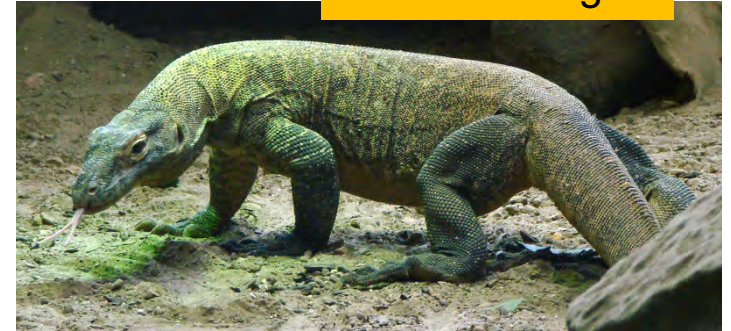
Size and Posture

How to Train Your Dragon (2010)



These dragons are about the size of an iguana so their posture is similar to that of an iguana.

Komodo Dragon



Iguana



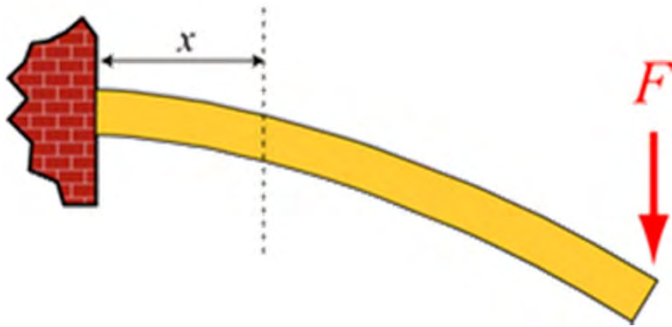
Anole Lizard



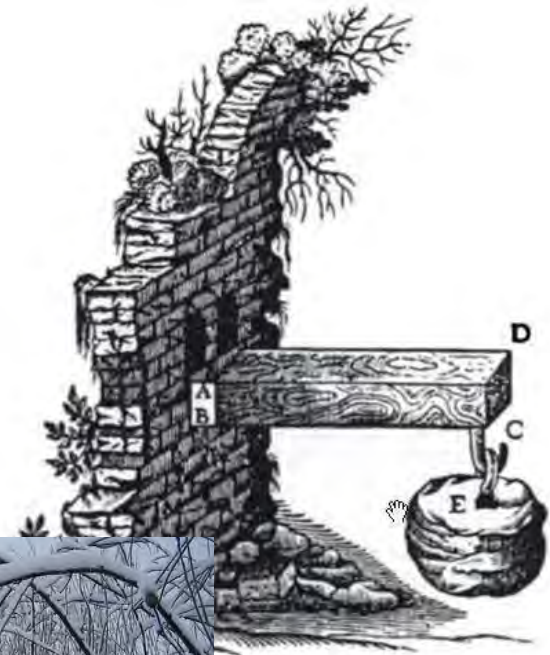
Bending Strength

Strength goes as area;
weight goes as volume.

Bending strength is a
physical cue for size.



Large beams bend
more, relative to their
size, while small beams
are relatively stiff.



Galileo pointed
out this scaling
law for beams.

Relative Stiffness

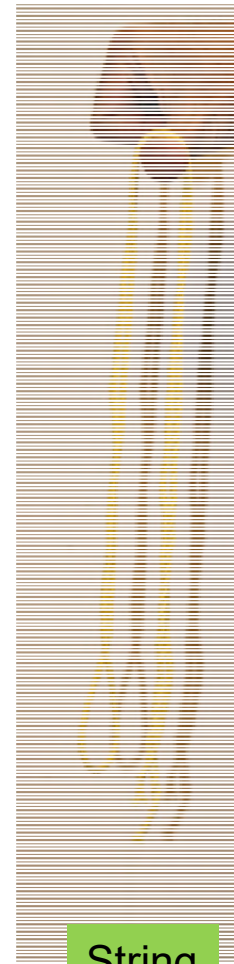
Stiffness goes as area;
weight goes as volume.

Relative stiffness is a
physical cue for size.

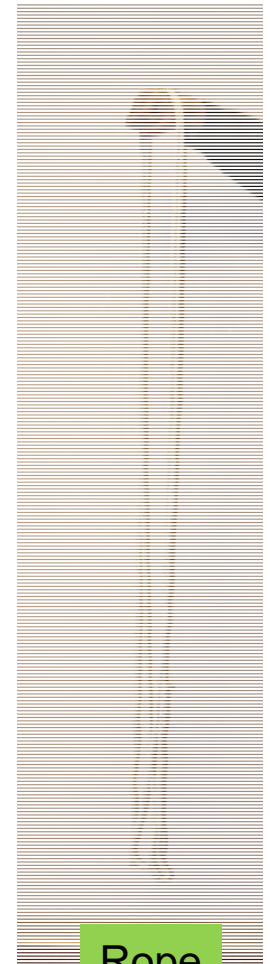
Thread, string, and rope
differ by about x4 in size
(length and diameter)
and they are made of
similar materials.



Thread



String



Rope

Squash Deformation

Squash deformation due to weight is a physical cue for size.

Surface forces vary with area but weight goes as volume

Small drops are spherical while large drops are flattened.



Harbor seal



Elephant seal



Burning and Dissolving

The time it takes to burn or to dissolve is a physical cue for size.

Burn or dissolve rate goes as area but total amount goes as volume.

Kindling is consumed quickly while large logs take longer to burn.



Cubes dissolve slower than grains.



Summary

- Area and volume both increase with size but at different rates with volume increasing faster.
- Weight goes as volume while strength (bone and muscle) goes as area.
- Large animals have proportionally thicker bones and stand straighter to support their weight.
- The larger the object the more it tends to sag proportionally under its own weight.
- The larger the object the longer it takes for it to burn or to dissolve.